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| 3rd Order square matrices |
| 3.3.1 |
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3rd order Matrices

Up until now we have explored how 2ndorder matrices can be used to solve simultaneous equations having two unknown variables. The key to using matrices is the inverse matrix. Inevitably, the natural progression would be to apply this method to solving simultaneous equations with 3 unknown variables. As mentioned earlier, matrices are a highly structured method of expressing conventional equations, but in this format, ideally suited to a PC based programming solution.

Unfortunately, there are more steps involved compared to the use of 2nd order matrices. However each step is simple, but in some instances repeated many times with only limited variation. In a short space of time, it will be easy to appreciate that a PC application is truly at home making these calculations without error.

Step 1

Find the determinant of the 3rd order square matrix:



Det│A =





a.(e.i – h.f) – b(d.i – g.f) + c(d.h – g.e) = Det│A

The process begins with finding the *minor* of the elements a,b and c. This is achieved by not including the elements directly vertical or horizontal with the positions a,b or c.



So the element ‘a’ has a *minor* of e,f,h and i. Treating this exactly in the same way as a 2nd order matrix, i.e. multiplying and subtracting the diagonal elements, finally we multiply the result by the element itself. You may have noticed that the result for the second minor is subtracted from the first, but the third minor is added. For the process to work, + or – is assigned to each element in its location and naturally effects the final *signed* outcome.

The following matrix shows the signed layout,



An alternating pattern of +, - is configured, starting with + in the top left hand element. We will need to apply this to each element *minor*. The determinant value is later used as a denominator for the inverse of the matrix.

Step 2

For the entire elements for the matrix, a *cofactor* for each element is found and replaces the original element to create a matrix of cofactors. The following diagram gives a clear idea:



Element ‘a’ is now replaced by the result of (e.i – h.f), Element ‘b’ is now replaced by the result of (d.i – g.f) and so on. However, the result must be signed by the +, - corresponding to the elements location, as shown the signed matrix above.

The resulting matrix is now termed the matrix of *cofactors*.

The next simple part of the process is to transpose the matrix. The elements in rows are repositioned as columns. The next matrix shows the transposed elements of an original matrix,



We can see the original top row of elements now form the first column of elements. The second row of elements is now the second column of elements and so on.

It is an important fact to point out that the determinant value of a transposed matrix *remains unchanged*. However, for the process to be successful, a transposition is necessary.

This matrix in this form i.e. cofactors and transposed is now termed the *Ajoint* of a square matrix

adj│A

or CT

C denotes cofactor and T Transpose

We now have the inverse of the matrix A­¹



The use of the 3rd order inverse matrix in solving simultaneous equations follows the same process as 2nd order matrices. The inverse matrix multiplies both sides of the equation, cancelling out and producing a *unit* matrix one side and producing a result the other. The rule for a unit matrix is exploited and enables the final calculation.

Try this example:

P+2Q-R = 4

3P-Q+R = 2

P+Q+R = 4

Answer: P=1, Q=2, R=1

Gaussian elimination Method

Finally, we will look at an alternative method to solve simultaneous equations in matrix form.

Consider:



For this example it will be better demonstrated using actual figures:

x + 2y -3z = 3

2x – y - z = 11

3x+2y +z = -5

Expressed in matrix form:



Subtracting 2x times the value of the elements of the first row from the second and 3x times the first row from the third will produce:



Next, we can multiple the second row by 4/5 and subtract it from the third row, this produces:



Now, by substitution and beginning with the third row,

6z = -18

z= -3

Substituting z i.e. -3 into the second row,

-5y -3.(5) =5

-5y -15 =5

y = -4

and finally, substituting both z and y into the first row

x+(-4).2-3.(-3) = 3

x-8+9 = 3

x=2

So; x=2, y=-4 and z=-3.

A much quicker method compared to a conventional matrix solution, but it may not lend itself to a PC formatted solution. When working in this manner, we can:

1. Interchange two rows
2. Multiple any row by a non-zero factor
3. Add or subtract a multiple of any row from another.

Self assessment question

X-4y-2z = 21

2x+y+2z = 3

3x+2y\_z = -2

Answer x= 3 y = -5 z = 1